Characteristics of Coupled Microstriplines

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Abstract—**Semiempirical design equations for the even- and odd-mode characteristics of coupled microstriplines are presented. The characteristics include capacitance, effective dielectric constant, impedance, and losses. The coupled line capacitances are obtained by suitably dividing the total capacitance into parallel plate and fringing capacitances. These capacitances are then used to determine other characteristics. The accuracy of characteristic impedances obtained from these capacitances is better than 3 percent. The sensitivity of the characteristics of coupled microstriplines to the tolerances in parameters is described. It is observed that the effect of tolerances on the coupling constant of a directional coupler increases with the increase in the value of coupling constant. The effects of dispersion and the finite thickness of metal strips have been included. It is noticed that the dispersion is more pronounced for even mode, whereas finite strip thickness affects odd mode to a larger extent.**

I. INTRODUCTION

COUPLED microstriplines are used in a number of circuit functions. The principal application areas are directional couplers, filters, and delay lines. There are numerous papers dealing with the analysis, design, and applications of coupled microstriplines. The analyses have been reported using various techniques. These are the even- and odd-mode method [1], the coupled-mode formulation [2], and the congruent-transformation technique [3].

The properties of coupled lines are determined by the self- and mutual inductances and capacitances between the lines. Under quasi-TEM approximation, the self-inductance can be expressed in terms of self-capacitance by using a simple relation. It is also found that for most of the practical circuits using symmetric coupled microstriplines, the mutual inductance and the capacitance are interrelated, and it is not necessary to determine the mutual inductance separately [2]. Therefore, only capacitance parameters are evaluated for coupled microstriplines. The numerical techniques used for evaluating these parameters are Green's-function method [4], variational method [5], [6], and Fourier-series expansion method [7]. Conformal-mapping technique has been reported by Pregla [8].

Recently, a number of papers have reported the usage of a single microstripline as an intermediate step for designing coupled microstriplines [9]–[11]. These authors succeed only partially in their objective. Shamanna *et al.* [9] have designed nomograms which can provide only rough accuracy and cannot be incorporated in the computer-aided design. Shamasundara and Singh [10] have given a scaling procedure for determining even- and odd-mode impedances which requires, for its usage, the accurate reference values for any other dielectric constant. Akhtarzad *et al.* [11] have provided a synthesis procedure for the design of a coupled microstripline. These expressions have an error of the order of 10 percent. Design equations have been provided by Ros [12]. These equations are inaccurate by 22 percent when compared with the results of Bryant and Weiss [13].

The approach used in this paper is to represent the total capacitance of a line to ground in terms of a parallel plate capacitance and two fringing capacitances, one for each side of the strip. This technique has been used successfully for coupled striplines [14] but the microstripline has not yielded to this approach because of its inhomogeneous dielectric configuration. Also attempts have been made in the past [15] to use this technique for coupled microstriplines but it resulted in the expressions which are very approximate. In this paper, capacitance expression for single mcirostripline has been utilized to determine the even-mode fringing capacitances. The odd-mode fringing capacitances are determined with the help of an equivalent geometry for coupled striplines and coplanar strips. The capacitance expressions are then used to determine characteristic impedances and effective dielectric constants.

II. CHARACTERISTICS

A. Even- and Odd-Mode Capacitances

The coupled microstriplines geometry is shown in Fig. 1. The breakup of line capacitance into parallel plate and fringing capacitances is also shown. Using these capacitances the total even- and odd-mode capacitances may be written as

$$C_e = C_p + C_f + C_f' \qquad (1)$$

$$C_o = C_p + C_f + C_{ga} + C_{gd} \qquad (2)$$

where

$$C_p = \epsilon_0 \epsilon_r W/h. \qquad (3)$$

C_f, C_f', C_{ga}, and C_{gd} represent various fringing capacitances. C_f is the fringing capacitance of a microstripline of width W/h, impedance Z_0, and effective dielectric constant ϵ_{re}, and is given by

$$2C_f = \sqrt{\epsilon_{re}}/cZ_0 - C_p, \quad c = 3 \times 10^8 \text{ m/s}. \qquad (4)$$

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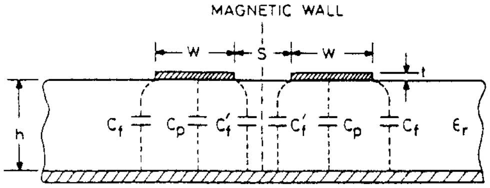
(a)

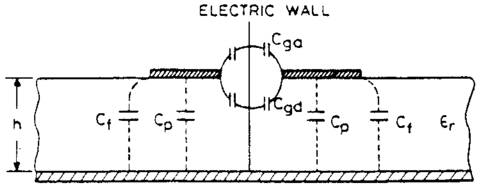
(b)

Fig. 1. Decomposition of total capacitance of coupled microstriplines in terms of various capacitances. (a) Even-mode capacitances. (b) Odd-mode capacitances.

The expression for capacitance C_f' is obtained empirically, such that the resulting value of even-mode capacitance compares with numerical results. The expression for C_f' is obtained as

$$C_f' = \frac{C_f}{1+A(h/s)\tanh(8S/h)} \quad (5)$$

where

$$A = \exp\left[-0.1\exp(2.33-2.53\,W/h)\right]. \quad (6)$$

C_{ga} is the capacitance term in odd mode for the fringing field across the gap, in air region. It is obtained from an equivalent geometry of coplanar strips, and is given by

$$C_{ga} = \epsilon_0 \frac{K(k')}{K(k)} \quad k = \frac{S/h}{S/h+2W/h} \quad k' = \sqrt{1-k^2} \quad (7a)$$

where the ratio of the complete elliptic function $K(k)$ and its complement $K(k')$ is given by

$$\frac{K(k)}{K(k')} = \begin{cases} \dfrac{1}{\pi}\ln\left[2\dfrac{1+\sqrt{k'}}{1-\sqrt{k'}}\right], & 0 \leq k^2 \leq 0.5 \\ \dfrac{\pi}{\ln\left[2\dfrac{1+\sqrt{k}}{1-\sqrt{k}}\right]}, & 0.5 \leq k^2 \leq 1 \end{cases} \quad (7b)$$

C_{gd} represents the capacitiance in odd mode for the fringing field across the gap, in the dielectric region. It is evaluated by modifying the corresponding capacitance for coupled striplines as follows:

$$C_{gd} = \frac{\epsilon_0\epsilon_r}{\pi}\ln\left\{\coth\left(\frac{\pi}{4}\frac{S}{h}\right)\right\} + 0.65C_f\left[\frac{0.02}{S/h}\sqrt{\epsilon_r} + 1 - \epsilon_r^{-2}\right]. \quad (8)$$

B. Characteristic Impedances and Effective Dielectric Constants

The characteristic impedances and effective dielectric constants for the two modes can be obtained from the capacitance values, using the following relations:

$$Z_{0i} = c\left[\sqrt{C_i C_i^a}\right]^{-1} \quad (9)$$

and

$$\epsilon_{re}^i = C_i / C_i^a \quad (10)$$

where i stands for even or odd mode, and C^a denotes the capacitance with air as dielectric. Comparison of calculated impedance values with the numerical results of Bryant and Weiss [16] shows an agreement better than 3 percent for $0.2 \leq W/h \leq 2$; $0.05 \leq S/h \leq 2$, and $\epsilon_r \geq 1$. It may be pointed out that this range of parameters represents a value of coupling constant $C \leq 0.6$. A typical set of calculations for $\epsilon_r = 9.6$ are compared in Table I with the results of Bryant and Weiss [16] and Coats [17]. The accuracy of guide wavelength has been found to be about 2 percent. To the authors' knowledge the design equations presented above are the most accurate so far available in the published literature.

The capacitance expressions given above are valid for coupled microstriplines with zero strip thickness. In actual practice, the metal strips have finite thickness. In addition, the effect of dispersion, parameter tolerances, and the finite amount of losses should also be considered while designing a coupled line device. These effects are considered next.

Effect of Strip Thickness: When the strip conductors are of finite thickness t, capacitances can be evaluated by using the concept of effective width, as enunciated by Wheeler for single microstripline [18].[1] An expression for effective width W_t has been obtained by Jansen [19] by modifying the corresponding expression for single microstripline. These expressions, valid for $S \geq 2t$, are reproduced below:

$$\frac{W_t^e}{h} = \frac{W}{h} + \frac{\Delta W}{h}\left[1 - 0.5\exp(-0.69\Delta W/\Delta t)\right] \quad (11)$$

$$\frac{W_t^o}{h} = \frac{W_t^e}{h} + \frac{\Delta t}{h} \quad (12)$$

where

$$\frac{\Delta t}{h} = \frac{1}{\epsilon_r}\frac{t/h}{S/h} \quad (13)$$

and ΔW is the increase in strip width of single microstripline due to strip thickness t [19]. The excess increase in effective width for the odd mode when compared with the even mode Δt, has been calculated by modeling the excess capacitance, over $t=0$ case, by a parallel-plate capacitance.

Due to the increase in even- and odd-mode capacitances with finite strip thickness, the even- and the odd-mode impedances decrease. The decrease in odd-mode impedance is about 2 percent for $t/h = 0.0047$, and

[1]This approximation appears more valid for the even-mode case for which the field configuration is somewhat similar to that for single microstripline. Nevertheless, it is also a reasonable practical approximation for the odd mode.

TABLE I
COMPARISON OF IMPEDANCE VALUES FOR $\epsilon_r = 9.6$

Dimensions		Bryant and Weiss [16] *		Coats [17]		This method		Coupling constant
W/h	S/h	Even	Odd	Even	Odd	Even	Odd	C
0.2	0.05	140.5	37.5	140	36	138	38.3	0.565
0.2	0.2	129.2	53.4	128.5	52.0	130.0	52.9	0.423
0.2	0.5	116.5	67.0	115.0	67.0	117.0	66.6	0.273
0.2	1.0	104.9	78.7	104.5	77.5	107.0	79.0	0.151
0.5	0.05	97.1	29.6	97.0	28	96.3	30.8	0.515
0.5	0.2	92.2	39.9	91.5	39.0	92.6	41.2	0.385
0.5	0.5	84.5	49.7	84.0	49.0	85.2	50.1	0.259
0.5	1.0	77.4	57.7	77.0	57.7	78.9	57.9	0.154
1.0	0.05	67.5	24.7	67.5	23.5	66.3	24.9	0.454
1.0	0.2	64.5	31.6	64.5	31.5	64.7	32.3	0.334
1.0	0.5	60.5	38.1	60.0	38.0	60.8	38.1	0.230
1.0	1.0	56.5	43.2	57.0	42.0	57.3	42.8	0.145
2.0	0.05	42.4	19.6	42.0	18.0	41.8	19.1	0.373
2.0	0.2	41.0	23.7	41.0	23.0	41.1	23.8	0.267
2.0	0.5	39.4	27.4	39.0	27.0	39.4	27.1	0.186
2.0	1.0	37.6	30.1	37.5	29.5	37.8	29.6	0.121

* The results of Bryant and Weiss for $\epsilon_r = 9.0$ are scaled to $\epsilon_r = 9.6$ by assuming that the impedance varies as $(\epsilon_r + 1)^{-\frac{1}{2}}$, as shown in [10].

$\epsilon_r = 9.6$. It is negligible for even mode. It is seen that the percentage increase in C_o^a (and C_e^a) with thickness is more than that in C_o (and C_e). Thus effective dielectric constants $\epsilon_{re}^e(t)$ and $\epsilon_{re}^o(t)$ decrease with thickness. The percentage decrease in ϵ_{re}^o is found to be more than that in ϵ_{re}^e because of an additional gap capacitance $2\epsilon_o t/s$, with air as dielectric.

Effect of Dispersion: The dispersive behavior of coupled microstriplines has been well described using numerical methods [19]–[22]. Semiempirical expressions by Getsinger [23] and Carlin and Civalleri [24], which describe dispersion in effective dielectric constant, are very similar. But the results obtained by using Getsinger's formula are closer to experimental values and are widely used. This expression may be written as

$$\epsilon_{re}^i(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{re}^i}{1 + G(f/f_p)^2} \tag{14}$$

where

$$G = \begin{cases} 0.6 + 0.018\, Z_{0o}, & \text{for odd mode} \\ 0.6 + 0.0045\, Z_{0e}, & \text{for even mode} \end{cases} \tag{15}$$

and

$$f_p = \begin{cases} 31.32\, Z_{0o}/h, & \text{for odd mode} \\ 7.83\, Z_{0e}/h, & \text{for even mode.} \end{cases} \tag{16}$$

Here f_p is defined in GHz, and h is in mil.

The design equations for the frequency dependence of even- and odd-mode impedances are not available. We have found that an equation similar to (14) can be written for impedances also. It is given below:

$$Z_{0i}(f) = Z_{Ti} - \frac{Z_{Ti} - Z_{0i}}{1 + G(f/f_p)^{1.6}} \tag{17}$$

where G and f_p are given by (15) and (16) and Z_{0i} represents the quasi-static value of coupled microstriplines impedance. Z_{Ti} is the corresponding impedance for coupled striplines with the same values of S and W as for coupled microstriplines. But the spacing between the two ground planes is $2h$. Values for Z_{Ti} are obtained from the following relations [14]:

$$Z_{Ti} = \frac{60\pi}{\sqrt{\epsilon_r}} \frac{K(k_i)}{K(k_i')} \tag{18}$$

$$k_i = \begin{cases} \tanh\left(\frac{\pi}{4}\frac{W}{h}\right) / \tanh\left(\frac{\pi}{4}\frac{W+S}{h}\right), & \text{for odd mode} \\ \tanh\left(\frac{\pi}{4}\frac{W}{h}\right) \tanh\left(\frac{\pi}{4}\frac{W+S}{h}\right), & \text{for even mode.} \end{cases} \tag{19}$$

The ratio $K(k_i)/K(k_i')$ is defined in (7b).

The results for dispersion in impedances calculated using (17) are compared with numerical values of Jansen [19] in Table II. It is observed that the agreement is quite good. Also, the increase in Z_{0e} is about 5 percent at 7.5 GHz.

C. Losses

Coupled microstriplines have two types of losses: ohmic and dielectric. The even- and odd-mode attenuation con-

TABLE II
INCREASE IN THE CHARACTERISTIC IMPEDANCES OF COUPLED MICROSTRIPLINES DUE TO DISPERSION; $\epsilon_r=9.9$, $W/h=0.945$

S/h	f(GHz)	Even mode Jansen [19]	Even mode Eqn.(17)	Odd mode Jansen [19]	Odd mode Eqn.(17)
	0	67.5	67.5	26.5	26.5
0.1	7.5	70.0	69.9	27.0	27.7
	15.0	74.4	73.2	28.4	29.6
	0	61.3	61.3	38.0	38.0
0.5	7.5	63.7	63.7	38.6	39.0
	15.0	67.2	66.7	40.7	40.8
	0	53.5	53.5	47.5	47.5
2.0	7.5	55.1	56.0	48.6	48.4
	15.0	57.8	59.1	51.4	50.0

stants due to ohmic losses in coupled microstriplines can be determined using incremental inductance rule of Wheeler [25]. Its application to the coupled line configuration yields the following expression for the odd-mode attenuation constant:

$$\alpha_c^o=\frac{8.686\,R_s}{240\pi Z_{0o}}\frac{2}{h}\frac{1}{c\left(C_o^{at}\right)^2}\cdot\left[\frac{\partial C_o^{at}}{\partial(W/h)}\left(1+\delta\frac{W}{2h}\right)-\frac{\partial C_o^{at}}{\partial(S/h)}\left(1-\delta\frac{S}{2h}\right)+\frac{\partial C_o^{at}}{\partial(t/h)}\left(1+\delta\frac{t}{2h}\right)\right],\quad \text{dB/unit length.} \quad (20)$$

Similarly, for the even mode

$$\alpha_c^e=\frac{8.686\,R_s}{240\pi Z_{0e}}\frac{2}{h}\frac{1}{c\left(C_e^{at}\right)^2}\cdot\left[\frac{\partial C_e^{at}}{\partial(W/h)}\left(1+\delta\frac{W}{2h}\right)-\frac{\partial C_e^{at}}{\partial(S/h)}\left(1-\delta\frac{S}{2h}\right)+\frac{\partial C_e^{at}}{\partial(t/h)}\left(1+\delta\frac{t}{2h}\right)\right],\quad \text{dB/unit length} \quad (21)$$

where

$$\delta=\begin{cases}1, & \text{for strips only}\\ 2, & \text{for strips and ground plane.}\end{cases}$$

C_o^{at} and C_e^{at} represent odd- and even-mode line capacitances, respectively, for air as dielectric medium and with finite thickness of strip. R_s is the sheet resistivity of metallization. Conductor losses have been calculated using (20) and (21). Comparison with the results of Rao [26] for losses in strips only, as given in Table III, indicates an agreement within 1 dB for odd mode. The agreement is better for even mode.

The attenuation due to dielectric loss α_d is given by [26]

$$\alpha_d^e=27.3\frac{\epsilon_r}{\sqrt{\epsilon_{re}^e}}\frac{\epsilon_{re}^e-1}{\epsilon_r-1}\frac{\tan\delta}{\lambda_0},\quad \text{dB/unit length} \quad (22)$$

$$\alpha_d^o=27.3\frac{\epsilon_r}{\sqrt{\epsilon_{re}^o}}\frac{\epsilon_{re}^o-1}{\epsilon_r-1}\frac{\tan\delta}{\lambda_0},\quad \text{dB/unit length} \quad (23)$$

where $\tan\delta$ is the loss tangent of the dielectric substrate, and λ_0 is the free-space wavelength.

The total loss $\alpha_c+\alpha_d$, in coupled microstriplines is plotted in Fig. 2 as a function of S for $\epsilon_r=9.7$, $W/h=0.944$, and $f=8$ GHz. It is observed from this figure that the odd-mode attenuation constant is always higher than the even-mode value. Also, it is more sensitive to changes in spacing S between the lines than is, the even-mode value. Comparison with the results of Jansen [19] for the same set of parameters indicates that the shape of the loss curves is identical in the two cases. However, Jansen has reported higher loss which occurs presumably due to additional factors like substrate surface roughness and different type of current distribution. The loss due to surface roughness is of the order of 10 percent for the alumina substrates at X band [27].

The loss calculations are important in the design of filters and couplers [26]. As an approximation the insertion loss of a coupled-line device with an input impedance of 50 Ω can be taken to be the average of even- and odd-mode losses [28].

D. Effect of Tolerances

The effect of tolerances in parameters on the characteristics of coupled microstriplines can be evaluated by means of sensitivity analysis. This analysis is similar to the one carried out for microstripline and slotline [29]. This analysis has been applied to a coupled microstriplines directional coupler. The maximum change in coupling constant is found to be given by the following expression:

$$\frac{|\Delta C|_{max}}{C}=\left|\frac{\Delta W}{W}S_W^C\right|+\left|\frac{\Delta h}{h}S_h^C\right|+\left|\frac{\Delta S}{S}S_S^C\right|+\left|\frac{\Delta\epsilon_r}{\epsilon_r}S_{\epsilon_r}^C\right| \quad (24)$$

where

$$C=\frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}} \quad (25a)$$

TABLE III
COMPARISON OF CONDUCTOR LOSSES IN COUPLED MICROSTRIPLINES WITH THE RESULTS OF RAO [26]; $f=8$ GHz, $t/h=0.0047$

ε_r	W/h	S/h	α_c^e(dB/m) Rao[26]	α_c^e(dB/m) Eqn.(21)	α_c^o(dB/m) Rao[26]	α_c^o(dB/m) Eqn.(20)
10.4	0.785	0.304	3.2	3.5	7.2	7.0
9.5	0.870	0.212	2.8	3.2	8.8	7.8
9.5	0.870	0.162	2.8	3.2	10.1	9.1

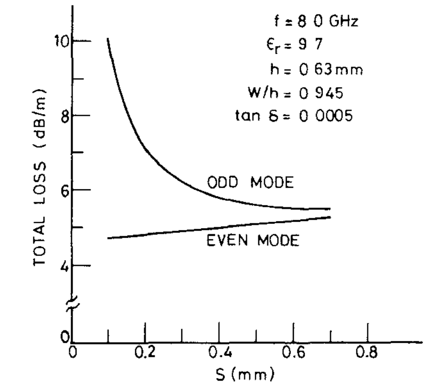

Fig. 2. Even- and odd-mode losses in coupled microstriplines.

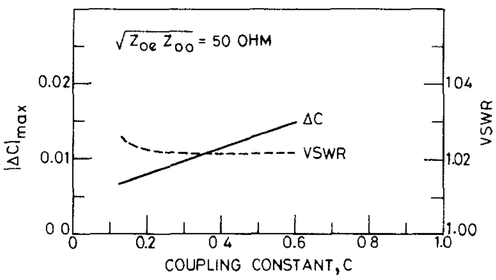

Fig. 3. Effect of tolerances on coupling constant and VSWR performance of coupled microstriplines directional couplers ($\epsilon_r=9.70\pm0.25$, $h=0.025\pm0.001$ in, $\Delta W=\pm0.0001$ in, and $\Delta S=\pm0.0001$ in.

and

$$S_B^A=\frac{B}{A}\frac{\partial A}{\partial B} \tag{25b}$$

and ΔW, Δh, ΔS, and $\Delta\epsilon_r$ are tolerances in parameters W, h, S, and ϵ_r, respectively. The expression for worst case VSWR due to the tolerances in parameters is given by

$$\text{VSWR}=\left[1-\frac{|\Delta Z_0|_{\max}}{Z_0}\right]^{-1} \tag{26}$$

where

$$Z_0=\sqrt{Z_{0e}Z_{0o}} \quad \text{and} \quad |\Delta Z_0|_{\max} \quad \text{is obtained from}$$

$$\frac{|\Delta Z_0|_{\max}}{Z_0}=\frac{1}{2}\left\{\left|\frac{\Delta W}{W}\left(S_W^{Z_{0e}}+S_W^{Z_{0o}}\right)\right|+\left|\frac{\Delta h}{h}\left(S_h^{Z_{0e}}+S_h^{Z_{0o}}\right)\right|+\left|\frac{\Delta S}{S}\left(S_S^{Z_{0e}}+S_S^{Z_{0o}}\right)\right|+\left|\frac{\Delta\epsilon_r}{\epsilon_r}\left(S_{\epsilon_r}^{Z_{0e}}+S_{\epsilon_r}^{Z_{0o}}\right)\right|\right\}. \tag{27}$$

Using (24)–(27) the effect of tolerances on the characteristics of a coupled microstriplines directional coupler can be estimated. It is shown in Fig. 3 for the change in coupling constant for $\epsilon_r=9.7$. The corresponding VSWR performance is also indicated therein. It may be observed from this figure that the effect of tolerances on the coupling constant increases with the increase in the value of C. However, the VSWR value remains almost constant. These results compare very well with the results of Shamasundara and Gupta [30].

IV. CONCLUSIONS

The even- and odd-mode impedances obtained from the semiempirical-design equations, presented in this paper, have been found to be accurate to within 3 percent for $\epsilon_r\geq1$, $0.2\leq W/h$, and $0.05\leq S/h\leq2$. These range of parameters represent a value of coupling constant ≤0.6. Losses in coupled microstriplines have been obtained using incremental inductance rule of Wheeler. It has been found that the variation of even- and odd-mode losses with gap spacing S is similar to the reported results. Even-mode loss compares to within 0.4 dB/m. However, the computed value of odd-mode loss is slightly lower. The effect of nonzero strip thickness and dispersion on the characteristics has been evaluated. It is noticed that the increase in the even-mode impedance due to dispersion is about 5 percent at 7.5 GHz. Sensitivity analysis has been carried out to calculate the worst case change in the characteristics of a directional coupler due to the tolerances in parameters. It has been found that for a directional coupler with 50-Ω input impedance the change in coupling constant, due to tolerances, increases with the increase in value of C. However, VSWR value remains almost constant.

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Letters

Slot Coupling Between Uniform Rectangular Waveguides

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Abstract—**The results of a recent paper, which analyzes the slot-coupled waveguide problem using a 'reaction' method, are shown to be at variance with those of more established theories, for the particular case of a centrally located transverse slot in the common broad wall separating a pair of rectangular waveguides.**

The boundary value problem comprising a pair of contiguous uniform rectangular waveguides connected electromagnetically by an aperture in the common wall, is a classical problem which has received considerable attention in the literature [1]–[7]. In general, the increasingly elaborate methods of solution which are presented have enabled more complex geometries to be examined, and more accurate results to be achieved. The more recent techniques, which rely on variation [5], [6] and moment [7] methods, have evolved, and become popular, as a direct consequence of the increasing availability of high-speed digital computers.

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Calculations have been performed, using several of the above methods, of the coupling coefficient c_{10} (see Fig. 1), associated with a centrally located transverse slot in the common broad wall between a pair of identical rectangular waveguides. The results of these calculations are presented in Fig. 2.

For nonresonant slots ($l<0.4\lambda$), Bethe's small aperture theory [1], modified by the resonance correction suggested by Levy [8], is a well-established and reliable analytical tool. The variational method of Sangster [5], and the moment method of Vu Khac [7], are in good agreement with the Bethe predictions over this range of slot sizes. The measured polarizabilities of Cohn [9] have been employed in the Bethe calculations to achieve this measure of agreement.

The curve of c_{10} versus slot length generated using the quasi-static antenna method, due to Lewin [3], is in general agreement with the variational and moment-method results, except